31. **Answer (E):** Find the common denominator and replace the ab in the numerator with $a-b$ to get

$$\begin{aligned}\frac{a}{b}+\frac{b}{a}-ab &= \frac{a^2+b^2-(ab)^2}{ab}\\ &= \frac{a^2+b^2-(a-b)^2}{ab}\\ &= \frac{a^2+b^2-(a^2-2ab+b^2)}{ab}\\ &= \frac{2ab}{ab}=2.\end{aligned}$$

OR

Note that $a = a/b-1$ and $b = 1-b/a$. It follows that $\frac{a}{b}+\frac{b}{a}-ab = (a+1)+(1-b)-(a-b) = 2$.

32. Answer: 112. Note that

$$AMC+AM+MC+CA = (A+1)(M+1)(C+1)-(A+M+C)-1 = pqr-13,$$

where p, q, and r are positive integers whose sum is 15. A case-by-case analysis shows that pqr is largest when $p=5$, $q=5$, and $r=5$. Thus the answer is $5\cdot 5\cdot 5-13 = 112$.

33. Answer: 5 Suppose that the whole family drank x cups of milk and y cups of coffee. Let n denote the number of people in the family. The information given implies that $x/4+y/6 = (x+y)/n$. This leads to

$$3x(n-4) = 2y(6-n).$$

Since x and y are positive, the only positive integer n for which both sides have the same sign is $n=5$.

OR

If Angela drank c cups of coffee and m cups of mile, then $0 < c < 1$ and $m+c=1$. The number of people in the family is $6c+4m = 4+2c$, which is an integer if and only if $c=\frac{1}{2}$. Thus, there are 5 people in the family.

34. Answer: 20. If x were less than or equal to 2, then 2 would be both the median and the mode of the list. Thus $x>2$. Consider the two cases $2<x<4$, and

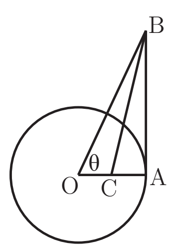
$x \geq 4$.
Case 1: If $2 < x < 4$, then 2 is the mode, x is the median, and $\frac{25+x}{7}$ is the mean, which must equal $2-(x-2)$, $\frac{x+2}{2}$, or $x+(x-2)$, depending on the size of the mean relative to 2 and x. These give $x = \frac{3}{8}$, $x = \frac{36}{5}$, and $x = 3$, of which $x = 3$ is the only value between 2 and 4.
Case 2: If $x \geq 4$, then 4 is the median, 2 is the mode, and $\frac{25+x}{7}$ is the mean, which must be 0,3, or 6. Thus $x = -25$, -4, or 17, of which 17 is the only one of these values greater than or equal to 4.
Thus the x-values sum to $3+17=20$.

35. **Answer (B):** Let $x = 9z$. Then $f(3z) = f(9z/3) = f(3z) = (9z)^2 + 9z + 1 = 7$. Simplifying and solving the equation for z yields $81z^2 + 9z - 6 = 0$, so $3(3z+1)(9z-2) = 0$. Thus $z = -1/3$ or $z = 2/9$. The sum of these values is $-1/9$.
Note. The answer can also be obtained by using the sum-of-roots formula on $81z^2 + 9z - 6 = 0$. The sum of the roots is $-9/81 = -1/9$.

36. Answer: 555. Suppose each square is identified by an ordered pair (m, n), where m is the row and n is the column in which it lies. In the original system, each square (m, n) has the number $17(m-1)+n$ assigned; in the renumbered system, it has the number $13(n-1)+m$ assigned to it. Equating the two expressions yields $4m - 3n = 1$, whose acceptable solutions are $(1,1)$, $(4,5)$, $(7,9)$, $(10,13)$, and $(13,17)$. These squares are numbered 1, 56, 111, 166 and 221, respectively, and the sum is 555.

37. **Answer (D):** The fact that $OA = 1$ implies that $BA = \tan\theta$ and $BO = \sec\theta$. Since $\overline{BC}$ bisects $\angle ABP$, it follows that $\frac{OB}{BA} = \frac{OC}{CA}$, which implies $\frac{OB}{OB+BA} = \frac{OC}{OC+CA} = OC$. Substituting yields

$$OC = \frac{\sec\theta}{\sec\theta + \tan\theta} = \frac{1}{1+\sin\theta}.$$

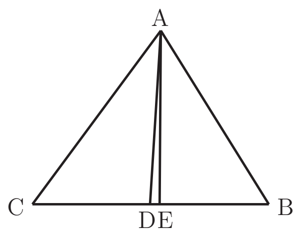


OR

Let $\alpha = \angle CBO = \angle ABC$. Using the *Law of Sines* on triangle BCO yields $\frac{\sin\theta}{BC} = \frac{\sin\alpha}{OC}$, so $OC = \frac{BC\sin\alpha}{\sin\theta}$. In right triangle ABC, $\sin\alpha = \frac{1-OC}{BC}$. Hence $OC = \frac{1-OC}{\sin\theta}$. Solving this for OC yields $OC = \frac{1}{1+\sin\theta}$.

38. Answer: Thursday. Note that, if a Tuesday is d days after a Tuesday, then d is a multiple of 7. Next, we need to consider whether any of the years $N-1$, N, $N+1$ is a leap year. If N is not a leap year, the 200$^{\text{th}}$ day of year $N+1$ is $365-300+200=265$ days after a Tuesday, and thus is a Monday, since 265 is 6 larger than a multiple of 7. Thus, year N is a leap year and the 200$^{\text{th}}$ day of year $N+1$ is another Tuesday (as given), being 266 days after a Tuesday. It follows that year $N-1$ is not a leap year. Therefore, the 100$^{\text{th}}$ day of year $N-1$ precedes the given Tuesday in year N by $365-100+300=565$ days, and therefore is a Thursday, since $565=7\cdot 80+5$ is 5 larger than a multiple of 7.

39. **Answer (B):** By Heron's Formula the area of triangle ABC is $\sqrt{(21)(8)(7)(6)}$, which is 84, so the altitude from vertex A is $2(84)/14=12$. The midpoint D divides $\overline{BC}$ into two segments of length 7, and the bisector of angle BAC divides $\overline{BC}$ into segments of length $14(13/28)=6.5$ and $14(15/28)=7.5$ (since the angle bisector divides the opposite side into lengths proportional to the remaining two sides). Thus the triangle ADE has base $DE=7-6.5=0.5$ and altitude 12, so its area is 3.



40. Answer: 1. Note that $(x+1/y)+(y+1/z)+(z+1/x)=4+1+7/3=22/3$ and that

$$\begin{aligned}28/3 = 4\cdot 1\cdot 7/3 &= (x+1/y)(y+1/z)(z+1/x)\\ &= xyz+x+y+z+1/x+1/y+1/z+1/(xyz)\\ &= xyz+22/3+1/(xyz).\end{aligned}$$

It follows that $xyz+1/(xyz)=2$ and $(xyz-1)^2=0$. Hence $xyz=1$.

OR

By substitution,

$$4 = x+\frac{1}{y} = x+\frac{1}{1-1/z} = x+\frac{1}{1-3x/(7x-3)} = x+\frac{7x+3}{4x-3}.$$

Thus $4(4x-3) = x(4x-3)+7x-3$, which simplifies to $(2x-3)^2 = 0$. Accordingly, $x = 3/2$, $z = 7/3 - 2/3 = 5/3$, and $y = 1 - 3/5 = 2/5$, so $xyz = (3/2)(2/5)(5/3) = 1$.